Forecasting the Nigerian Exchange Group's All Share Index: application of ARIMA-GARCH hybrid models

Marshal Simon Ekpete[1]*, Juliet Ifechi Kenn-Ndubuisi[2], Ipeghan Iyo[3]
[1]*Nigerian British University, Asa, Abia State, Nigeria*
[2,] *Department of Banking and Finance, Rivers State University, Port Harcourt, Nigeria*
[3] *Department of Finance, Ignatius Ajuru University of Education, Port Harcourt, Nigeria*
**Corresponding author: ekpete.marshal@nbu.edu.ng*
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Abstract

Purpose: This study evaluates the predictive accuracy of various econometric models in forecasting stock market returns within the Nigerian Exchange Group (NGX). Given the high volatility and structural breaks inherent in emerging African markets, the research investigates whether linear (ARIMA), volatility-sensitive (GARCH), or hybrid rolling-window frameworks provide the most robust tools for financial forecasting.
Methodology: Employing a quantitative design, the study analyses daily All-Share Index (ASI) closing prices (sourced from the NGX official database) from January 1, 2021, to August 30, 2025. The approach utilizes Box-Jenkins formalization for ARIMA parameters and GARCH (1,1) to account for volatility clustering, optimized via a 180-day rolling window. Stationarity was verified via ADF tests, with selection guided by the Akaike Information Criterion (AIC).
Results and conclusion: Empirical findings indicate that hybrid ARIMA-GARCH models significantly outperform standalone ARIMA and GARCH models in forecasting the NGX ASI. While linear models fail to account for significant ARCH effects, the hybrid configuration markedly reduces forecast errors, as confirmed by superior MAPE and RMSE metrics. The NGX exhibits high volatility persistence, rendering traditional linear models insufficient. Findings suggest the NGX deviates from the Weak-Form Efficient Market Hypothesis, as historical patterns retain predictive value. **Implication of findings:** These results advise institutional investors to prioritize volatility-aware models for asset pricing and Value-at-Risk (VaR) estimations. Furthermore, they underscore the need for policymakers to implement stabilizing interventions during periods of high turbulence.

Keywords: ARIMA-GARCH, Volatility clustering, Market forecasting, Hybrid models, Rolling window.

1. Introduction

The Nigerian capital market serves as a strategic cornerstone for national wealth mobilization, with market capitalization exceeding ₦101 trillion in the first quarter of 2026 (NGX, 2026). This significant growth has been catalyzed by the Investments and Securities Act (ISA) 2025, which established a modern regulatory framework for virtual assets, and the transition to a T+2 settlement cycle, aligning the Nigerian Exchange Group (NGX) with global standards (DCSL, 2025).

However, the NGX faces unique challenges that complicate price discovery. High inflation rates, which peaked above 30% in 2024, and the industry-wide recapitalization drive for banks, facing a strict March 2026 deadline, which 30 deposit money banks met, have introduced significant "noise" and structural breaks into the market (Coronation MB, 2026). These macroeconomic headwinds create an environment where the All-Share Index (ASI) exhibits both linear trends and intense, non-linear volatility clustering. Consequently, the ability to accurately forecast market indices is essential for stress testing and informed decision-making in a landscape characterized by frequent "persistence shocks."

The primary methodological challenge is that traditional forecasting models often prove insufficient in this context. Previous studies on the NGX have largely utilised standalone ARIMA models, which assume constant variance (homoscedasticity) and fail to capture the "memory" of market volatility (Ajibodu & Agunloye, 2021). As noted by Adhikari (2024), static models lose predictive power when they cannot account for the tendency of large price swings to follow one another. Furthermore, traditional Box-Jenkins methodologies are "atheoretic" and struggle in highly random environments where patterns do not remain constant (Adenomon & Emmanuel, 2024). This study addresses these gaps by implementing a hybrid ARIMA-GARCH framework augmented with a 180-day rolling window.

The unique contribution of this research lies in this dynamic rolling optimization. Unlike static models that rely on a fixed historical dataset, the rolling window techniques supported by recent research as more robust for fluctuating markets (JIHMSP, 2025), continuously updates the model parameters to incorporate the most recent market shocks while discarding obsolete data. The primary objective is to evaluate the efficacy of this hybrid model in predicting NGX ASI prices, specifically assessing its accuracy against traditional benchmarks and demonstrating its real-time adaptability. By bridging the gap between theoretical econometrics and practical risk management, this research provides a vital tool for navigating the complexities of Nigeria's evolving financial ecosystem.

2. Literature review

Theoretical review

The efficient market hypothesis (EMH): The foundational pillar of this study is the Efficient Market Hypothesis (EMH), famously refined by Fama (1965). EMH posits that in an informationally efficient market, stock prices instantaneously incorporate all available information, rendering technical or fundamental analysis incapable of consistently generating "alpha" (excess returns). In the context of the Nigerian Exchange Group (NGX), this study specifically investigates the Weak-Form of Efficiency. This level of efficiency asserts that the current price changes are independent of historical data. By employing ARIMA and GARCH models, this research essentially tests the "predictability" of the NGX. If these models demonstrate significant forecasting power, it suggests that the NGX deviates from the EMH, revealing market inefficiencies where historical patterns can be exploited for superior returns, a common characteristic of emerging economies with varying levels of information transparency.

The random walk theory: The study adopts the Random Walk Theory originated in 1900 with Louis Bachelier, who mathematically modelled stock prices as a stochastic process similar to Brownian motion, arguing that price changes are fundamentally unpredictable "fair games." Following statistical validation by Maurice Kendall (1953) and its integration into the Efficient Market Hypothesis by Eugene Fama (1960s), Burton Malkiel popularized the concept in his 1973 book, *A Random Walk Down Wall Street*. The random walk theory further suggests that stock price movements are stochastic and follow no discernible pattern or "memory." Mathematically, this implies that the best predictor of tomorrow's price is today's price plus an unpredictable random shock: $P_t = P_{(t-)} + e_t$. A key challenge in modelling the Nigerian stock market is that such series are typically non-stationary, meaning their mean and variance change over time. The theoretical justification for using the ARIMA (Autoregressive Integrated Moving Average) framework lies in its ability to handle this "random walk" behaviour. Through the process of differencing (d), the model transforms a non-stationary, unpredictable series into a stationary one, allowing for the extraction of underlying linear dependencies that would otherwise be obscured by the random walk.

The theory of volatility clustering (ARCH/GARCH): While previous theories focused on the direction of prices, the Theory of Volatility Clustering addresses the intensity of risk. Pioneered by Engle (1982) and expanded by Bollerslev (1986), this theory accounts for the "stylized facts" of financial time series, specifically, that volatility is not constant (heteroscedasticity) but occurs in cycles. In the Nigerian financial landscape, periods of high turbulence (often triggered by oil price shocks or policy shifts) tend to be followed by further instability, while periods of calm persist. Traditional ARIMA models are limited because they assume homoscedasticity (constant variance). The integration of the GARCH (Generalized Autoregressive Conditional Heteroscedasticity) model is therefore theoretically vital. It acknowledges the "memory" of risk, allowing the researcher to model the "clustering" of turbulence and provide a more accurate representation of the market's risk-return profile than linear models alone.

Empirical review

The following empirical review synthesizes recent literature on index forecasting, categorizing studies by their methodological complexity to highlight the specific gaps this research aims to bridge.

Adhikari (2024) serves as the primary methodological anchor for this research. In an evaluation of the NEPSE index, this study demonstrated that traditional static models fail to capture the fluid dynamics of emerging markets. By implementing a hybrid ARIMA (5,1,0)-GARCH (1,1) model augmented with a 180-day rolling window, the study achieved an exceptional correlation of 0.995. This research validates two critical premises of the current study: first, that GARCH models are essential to address ubiquitous volatility clustering, and second, that a rolling data window allows parameters to adaptively respond to structural breaks in emerging economies.

Recent literature increasingly favours such hybrid frameworks over standalone models when navigating the "noisy" environments of developing exchanges. For instance, while Ajibodu and Agunloye (2021) focused specifically on the Nigerian All-Share Index (ASI), their approach relied on purely linear ARIMA models, identifying ARIMA (1,2,5) as optimal. While accurate for in-sample data, their model lacked the "volatility awareness" provided by a GARCH component. In a similar vein of predictive refinement, Wang and Zhang (2022) compared traditional ARIMA with hybrid ARIMA-GARCH and ARIMA-SVM models across India and Brazil. Their findings mirrored Adhikari's, concluding that the integration of GARCH markedly improved long-term predictive power by accounting for conditional heteroscedasticity. This global validation is further supported by Dritsaki (2018) and Pahlavani and Roshan (2015), who applied hybrid ARIMA-GARCH models to oil prices and exchange rates, confirming that in volatile financial time series, the "memory" of past volatility is as predictive as the price trend itself.

Furthermore, this recent research suggests that even ARIMA-GARCH hybrids may have limitations in capturing the most complex market residuals. Kumar and Jha (2021) integrated ARIMA with Random Forests and Neural Networks for the Indian market, finding that while ARIMA was robust for linear trends, machine learning was required to "clean" non-linear residuals. This is supported by Amin and Aziz (2023), whose study of the S&P 500 and Nasdaq showed that while ARIMA (1,1,1) is an effective short-term forecaster, its performance degrades over longer horizons unless paired with volatility-aware frameworks. Additionally, Liu and Chen (2020) noted that for the Shanghai Stock Exchange, predictive accuracy is often capped by "Weak-Form Inefficiency," suggesting that a rolling window, as utilized in this study, is a necessary intervention to capture the rapid decay of historical information.

Ultimately, the consensus in recent literature, led by the high-accuracy results of Adhikari (2024), is that emerging market indices require models that are both volatility-aware and temporally adaptive. While previous Nigerian studies, such as Ajibodu and Agunloye (2021), have explored the ASI, there remains a significant research gap in applying a combined ARIMA-GARCH framework with a 180-day rolling optimization. This study addresses this gap by synthesizing these advanced econometric techniques to enhance real-time responsiveness to shocks in the Nigerian economy.

3. Methodology

This study adopts a quantitative research methodology through a multi-stage econometric approach, combining ARIMA linear modelling with a GARCH (1,1) volatility framework. Utilizing an ex post facto and longitudinal research design, the study analyses a population consisting of all listed equities on the Nigerian Exchange Group (NGX), specifically represented by daily All-Share Index (ASI) closing prices from January 1, 2021, to August 30, 2025. The GARCH (1,1) model was selected specifically due to its documented efficiency in capturing the "stylized facts" of financial time series with minimal parameters. While higher-order models exist, the (1,1) specification, incorporating one lag of squared error (ARCH term) and one lag of conditional variance (GARCH term), is widely recognized as the most robust and parsimonious configuration for modelling volatility clustering in emerging markets. It effectively avoids the risk of over-parameterization while remaining flexible enough to account for the enduring persistence of shocks characteristic of the NGX.

To ensure the integrity of the longitudinal analysis, a rigorous data-cleaning protocol was implemented to handle missing or anomalous data points. In instances of market holidays or technical trading halts where daily ASI values were missing, a linear interpolation method or the "last observation carried forward" (LOCF) approach was utilized to maintain the continuity of the 180-day rolling window without introducing artificial volatility. Furthermore, data points identified as outliers, those exceeding a four-standard-deviation threshold from the rolling mean, were scrutinized. If these were found to be the result of a data entry error rather than a fundamental market shock, they were smoothed using a three-day moving average to prevent the ARIMA-GARCH parameters from biased estimation or "exploding" variance.

The formal estimation procedures involve stationarity testing via Augmented Dickey-Fuller (ADF) and KPSS tests to determine the order of integration. This is followed by the Box-Jenkins formalization, which encompasses model identification, estimation, and diagnostic checking, alongside a 180-day rolling window optimization to ensure real-time responsiveness to market shifts. Within this framework, the independent variables include historical ASI prices and past volatility terms, while the dependent variables are the forecasted ASI values and the conditional variance, which serves as the primary metric for market risk at time t.

This study utilizes secondary time-series data consisting of the daily closing prices of the Nigerian Exchange Group (NGX) All-Share Index (ASI). The observation period spans from January 1, 2021, to August 30, 2025. To ensure data integrity and model robustness, the following pre-processing steps are implemented: (i) Data Cleaning, the dataset is audited for missing entries or artifacts. (i) with regards Outlier Detection, a Z-score analysis is conducted using a 180-day rolling window to identify and manage extreme fluctuations. (iii) in terms of Stationarity Testing, the Augmented Dickey-Fuller (ADF) test is employed to determine the integration order of the series. If non-stationarity is detected, first-order differencing (d=1) or higher is applied to achieve a stationary mean.

The ASI is a market capitalization-weighted index that serves as a barometer for the performance of all listed equities on the NGX. It is calculated as follows:

$$ASI= \frac{\sum_{I=1}^{n} (P_{it} \times Q_{it})}{\text{Base Value}} \times 100$$

Where: $\sum$ = summation symbol on the total listed firms; P_{it} = current market price of stock i at time t; Q_{it} = number of outstanding ordinary shares of company I at time t ; $\sum (P_{it} \times Q_{it})$ = Total Market capitalization of all listed ordinary shares l; Base value = market capitalization value as of January 1984

ARIMA model for linear patterns

To capture linear trends and temporal dependencies, the Autoregressive Integrated Moving Average (ARIMA) model is utilized. The model is expressed as:

$$\emptyset(L)(1 - L)^d y_t = \theta q(L)\epsilon t$$

Where: $\emptyset(L)$ represents the autoregressive (AR) part with p lags; $\theta(L)$ represents the moving average (MA) part with q lags; L is the lag operator: $L^k y_t = y_{t-k}$; D is the order of differencing to make the time series stationary; y_t is the time series at time t; $\in_t$ is white noise (error term)

The optimal (p, d, q) parameters are selected by minimizing the Akaike Information Criterion (AIC) through a stepwise procedure.

GARCH model for volatility clustering

Financial time series often exhibit volatility clustering, a phenomenon where "large changes tend to be followed by large changes, and small changes tend to be followed by small changes" (Mandelbrot, 1963). In the context of the Nigerian Exchange, this means that market stability or turbulence is rarely a fleeting event but rather occurs in sustained waves. After fitting the ARIMA model to capture linear trends, the residuals are tested for ARCH effects using the Lagrange Multiplier (LM) Test. If heteroscedasticity is detected, a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is specified to model the conditional variance (sigma_t^2):

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum^{p} \beta_j \sigma_{t-j}^2$$

Where : $\sigma 2$ is the conditional variance (volatility) at time t; $\alpha 0$ is a constant term; $\alpha i \epsilon 2$ captures the ARCH terms (impact of past squared residuals)l $\beta j \sigma 2$ captures the GARCH terms (impact of past variances); P is the number of lagged variances (GARCH terms); Q is the number of lagged squared residuals (ARCH terms).

With respect to the Significance of the coefficients, the alpha and beta coefficients provide deep insight into the "character" of market risk within the NGX: (i)The "news" Reaction (alpha), the alpha coefficient measures how intensely the market reacts to new information or sudden shocks. A high alpha suggests that the NGX is highly sensitive to recent events, such as a sudden policy announcement by the Central Bank. In such a market, volatility spikes sharply and immediately following a "shock." (ii)The Persistence or Memory known as the beta coefficient measures the "longevity" of volatility. A high beta indicates that once volatility enters the market, it takes a long time to dissipate. Even in the absence of new information, the market remains turbulent simply because it was turbulent in the previous period. (iii)The Persistence Sum (alpha and beta), is perhaps the most critical metric for risk assessment: IGARCH Behaviour: If the sum is close to 1, it indicates that volatility shocks are extremely persistent, while Market Impact implies in the context of the NGX, as a high alpha and beta sum, indicating that the impact of financial disruptions will "linger" in the system. This requires investors to maintain defensive positions and wider risk margins for longer periods following a market crisis.

The hybrid ARIMA-GARCH framework

The study adopts a two-stage hybrid approach to model both the conditional mean and conditional variance, including stage 1, the stationary series is fitted with the optimal ARIMA model to extract linear patterns. Then stage 2: the non-linear residuals from Stage 1 are modelled using GARCH to capture the volatility dynamics. This dual framework ensures that both the direction and the risk (volatility) of the NGX index are accounted for in the final forecast.

Optimization via rolling data window

To account for the dynamic and evolving nature of the Nigerian economy, a rolling window technique is implemented (Zivot & Wang, 2003). Rather than a static estimation, a fixed-size window of 180 trading days (approximately nine months of market activity) is utilized to maintain a balance between statistical depth and recent relevance.

The model follows a "Walk-Forward" validation process: at each step t, the ARIMA-GARCH parameters are re-estimated using the data within the [t−180,t] interval. As the window advances to t+1, the most recent data point is integrated, and the oldest is discarded. This constant recalibration allows the model's coefficients—specifically the ARCH (α) and GARCH (β) terms—to adapt to shifting market trends, policy changes, and structural breaks that a static model would otherwise smooth over.

Practical Challenges and Considerations While this dynamic adaptation enhances forecasting precision, it introduces specific implementation challenges: (i) Re-estimating non-linear GARCH parameters at every new data point is computationally intensive. In a real-time trading environment, this requires robust processing power and optimized algorithms to ensure that the "forecast horizon" does not expire before the model finishes converging. (ii)The NGX can be prone to "thin trading" or liquidity gaps. A 180-day window is susceptible to localized noise; if a significant portion of that window contains outliers or low-volume periods, the model may overfit to temporary volatility spikes rather than long-term structural shifts. (iii) Because the model is re-estimated iteratively, there is a risk that specific 180-day subsets may not meet the stationary requirements or local maximum likelihood constraints, leading to "non-convergence" of the GARCH parameters during automated runs. Researchers must implement fallback heuristics or "error-handling" parameters to maintain model stability during these iterations.

Model evaluation

The predictive performance and diagnostic health of the models are assessed using: Residual Diagnostics, Ljung-Box test for remaining autocorrelation, ARCH-LM test for remaining heteroscedasticity, and QQ-plots for normality. And Accuracy Metrics, the out-of-sample forecast accuracy is quantified using Mean Percentage Error (MPE), Root Mean Squared Error (RMSE), and Correlation Analysis between predicted and actual values.

4. Results and discussion

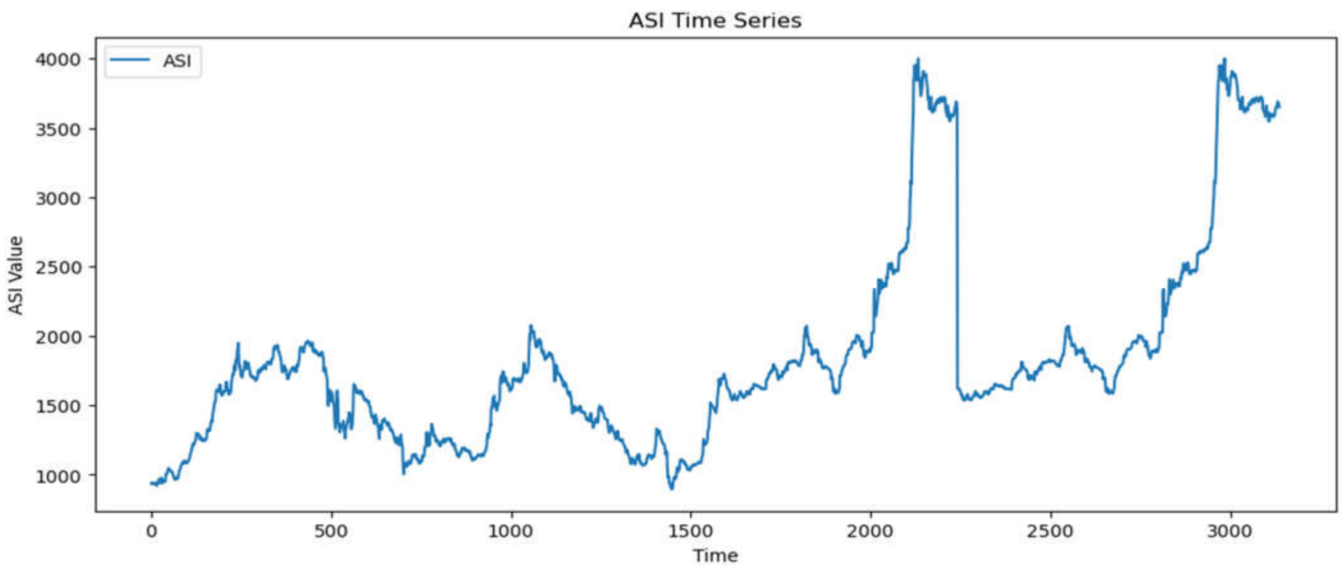

Figure 1: Time plot on observed values on ASI

Figure 1 displays a time plot of observed ASI series. The visual assessment of this plot indicates significant variations over time, highlighting intervals with both increased and decreased activity. This preliminary visualization supports evidence of non-stationarity within the series, which necessitates additional formal testing to identify the order of integration.

Table 1: Descriptive statistics of ASI

Statistic	Estimates
Mean	1836.43
Std. Dev	721.96
Skewness	1.565
Kurtosis	1.859
Minimum	895.10
Maximum	4000.79

Source: Authors computation (2026).

Table 1 encapsulates the statistical characteristics of the ASI series. The mean value reflects the central value of ASI, while the standard deviation reveals considerable variation in the observed data points. A positive skewness of 1.565 and notably low kurtosis of 1.859 suggest that the distribution is asymmetric, with a higher concentration of values on the lower end and fewer extreme outliers when compared to a normal distribution. These descriptive statistics indicate the necessity for ARIMA modeling to adequately capture trends and variations, as well as GARCH modeling to address conditional heteroskedasticity in volatility.

Table 2: Unit root test on ASI

Series	Test Type	Statistic	p-value	Conclusion
Level	ADF	-1.1811	0.6817	Non-stationary
	KPSS	3.7814	0.0100	Non-stationary
First Difference	ADF	-36.9575	0.0000	Stationary
	KPSS	0.0878	0.1000	Stationary

Source: Authors computation (2026).

Table 2 shows the findings from the ADF and KPSS unit root tests. Results from both tests indicate that the ASI series is non-stationary at levels, as demonstrated by the significance of the ADF statistic being absent and the KPSS test rejecting the null hypothesis of stationarity. Nonetheless, when the series is first differenced, it achieves stationarity, with both tests validating that the first-differenced series exhibits stationary characteristics. This result indicates that a differenced ARIMA model (d=1) is suitable for modeling the ASI series. The return series were examined, and the plot is shown in Figure 2 below.

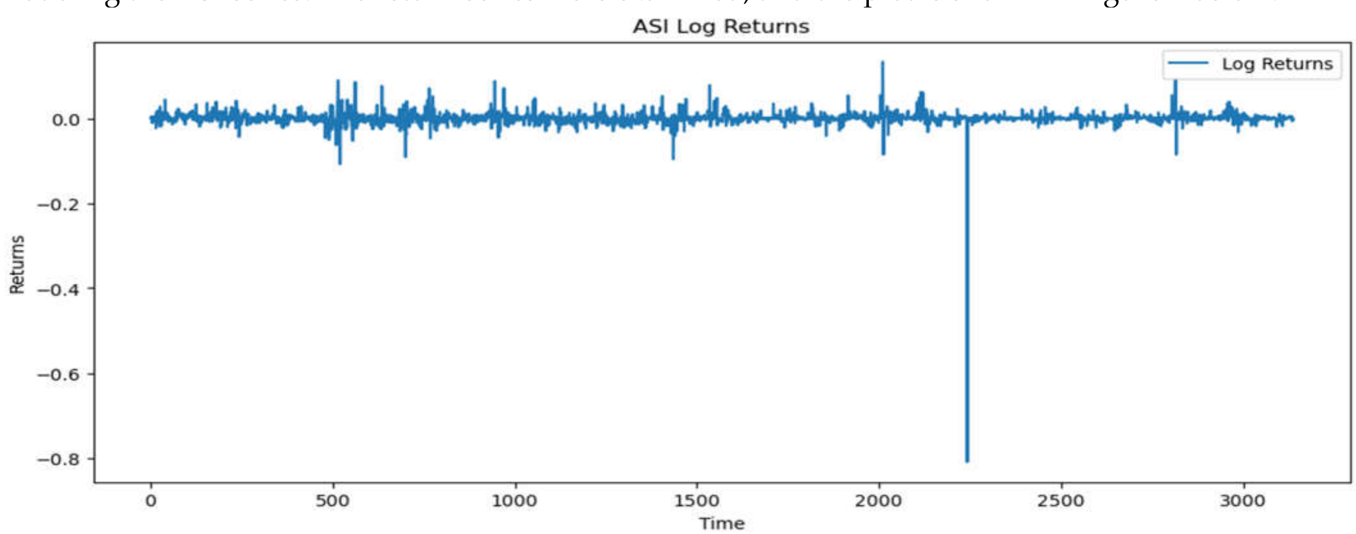

Figure 2: Time plot for returns on ASI

Visual examination of the plot in Figure 2 shows the Time Plot for Returns on ASI is stationary and there is evidence of volatility clustering

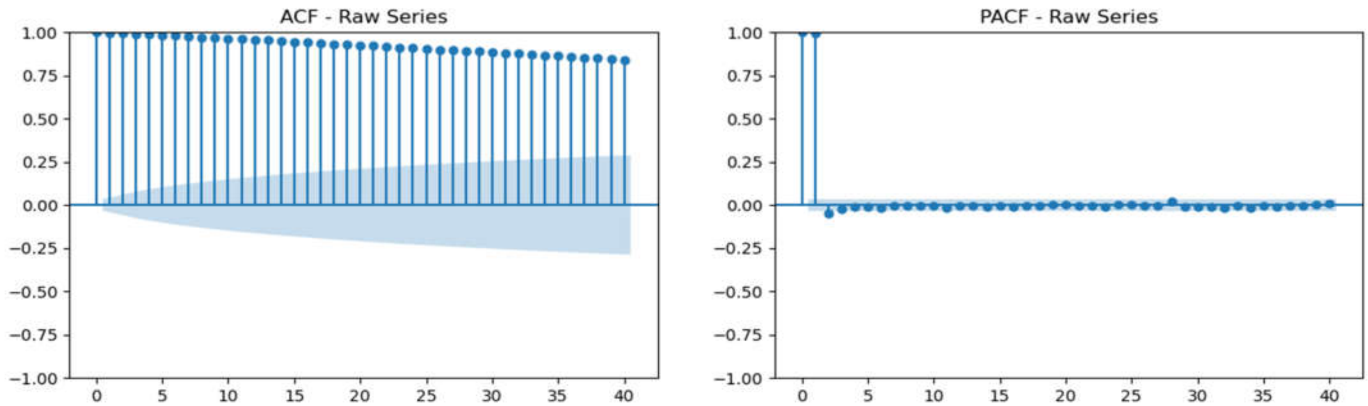

Figure 3: ACF and PACF plots on observed values on ASI

Figure 3 illustrates the ACF and PACF plots for the ASI series. Noticeable spikes in both ACF and PACF at early lags indicate the presence of autocorrelation within the series, supporting the need to incorporate both AR and MA elements into the ARIMA model. The patterns observed in these plots assist in selecting the appropriate ARIMA (1,1,1) configuration.

The ARIMA (1,1,1) model, estimated to use the first-differenced series, is expressed as:

$$\Delta y_t = 0.6404\Delta y_{t-1} - 0.5731\varepsilon_{t-1} + \varepsilon_t$$

or equivalently in levels:

$$y_t = y_{t-1} + 0.6404(y_{t-1} - y_{t-2}) - 0.5731\varepsilon_{t-1} + \varepsilon_t$$

This suggests that the present alterations in ASI are significantly affected by earlier modifications (AR coefficient) and prior disturbances (MA coefficient). The analysis of residuals (Figure 3) validates that the model effectively represents the linear characteristics of the series, as the residuals exhibit a random distribution.

Table 3: ARCH test and ARIMA model optimization results

Test / Info	Value / Details
ARCH Test on ARIMA Residuals	
F-statistic	0.0470
p-value	1.0000
ARIMA Model Optimization	
Iteration 5, Function Count 28	Neg. Log-Likelihood = 17725.19
Iteration 10, Function Count 55	Neg. Log-Likelihood = 16055.14
Iteration 15, Function Count 80	Neg. Log-Likelihood = 16017.10
Iteration 20, Function Count 105	Neg. Log-Likelihood = 16008.16
Optimization Status	Terminated successfully (Exit mode 0)
Current function value	16008.16
Total Iterations	21
Function evaluations	110
Gradient evaluations	21

Source: Authors computation (2026).

ARCH Test and ARIMA Optimization Table 3 are conducted on the residuals of the ARIMA model produces a p-value of 1.0000, signifying the absence of notable ARCH effects in the residuals. Nevertheless, due to the detected volatility clustering in the time series, a GARCH (1,1) model is utilized to more accurately reflect the conditional variance. The optimization of the ARIMA model shows convergence, with the negative log-likelihood stabilizing at 16008.16 after 21 iterations, confirming the reliability of the parameter estimates

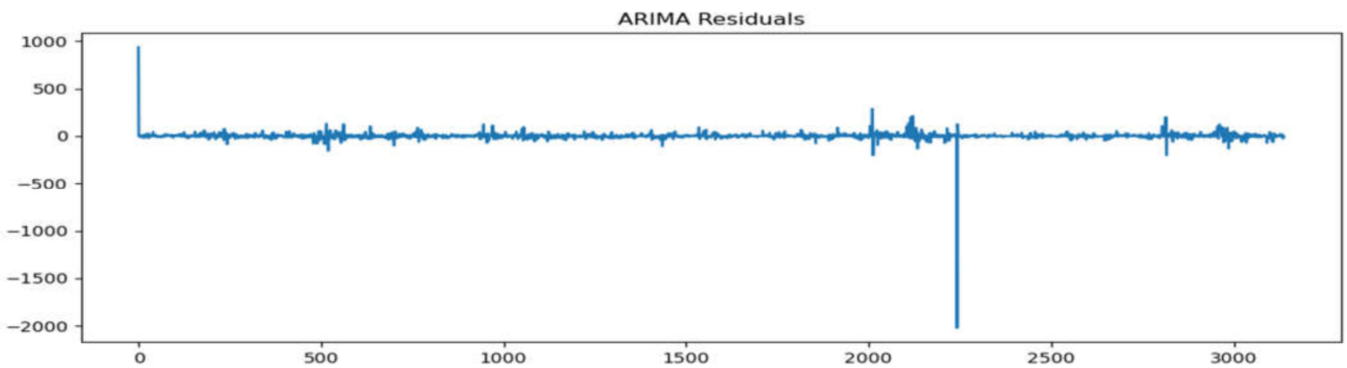

Figure 4: ARIMA residuals of the estimated ARIMA(1,1,1)

GARCH (1,1) Model Specification

Mean Equation (Constant Mean Model): $y_t = \mu + \varepsilon_t$

Substituting the estimated value: $y_t = 10.1647 + \varepsilon_t$

Variance Equation (GARCH(1,1)): $\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

Substituting the estimated coefficients: $\sigma_t^2 = 597.5870 + 0.6714\varepsilon_{t-1}^2 + 0.3286\sigma_{t-1}^2$

Full Model Representation $\varepsilon_t \sim N(0, \sigma_t^2)$

$$y_t = 10.1647 + \varepsilon_t$$

$$\sigma_t^2 = 597.5870 + 0.6714\varepsilon_{t-1}^2 + 0.3286\sigma_{t-1}^2$$

The results show that $\alpha_1 = 0.6714$ Strong impact of recent shocks (volatility clustering), $\beta_1 = 0.3286$: Persistence of past volatility , $\alpha_1 + \beta_1 = 1.0000$: Indicates high persistence (IGARCH-like behavior) , $\omega = 597.5870$: Long-run variance component and $\mu = 10.1647$: Mean return (not statistically significant at 5%). The equation $\alpha_1 + \beta_1 = 1.0000$ signifies a behavior akin to IGARCH, indicating that disturbances have a lasting influence on subsequent volatility. This suggests that times of significant ASI variability are likely to continue, which is essential for risk analysis and forecasting.

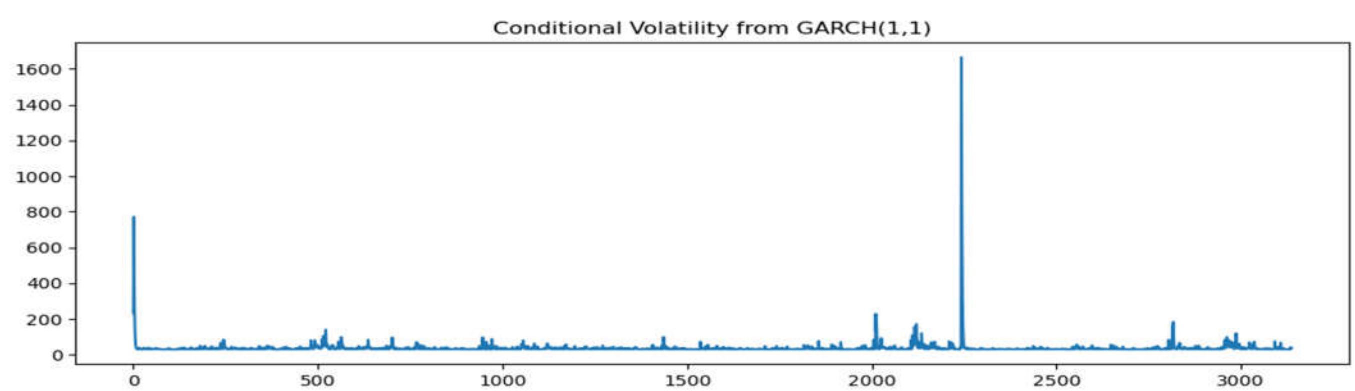

Figure 5: Plot of the conditional volatility from GARCH(1,1)

Figure 5 displays the conditional volatility obtained from the GARCH (1,1) framework. It depicts instances of increased risk, marked by volatility spikes during unstable periods. This depiction affirms the existence of volatility clustering, a prevalent feature in financial and economic time series data. The enduring effects of shocks, as indicated by the IGARCH-like nature, clarify why volatility does not quickly return to a long-term average but experiences extended phases of increased or reduced risk

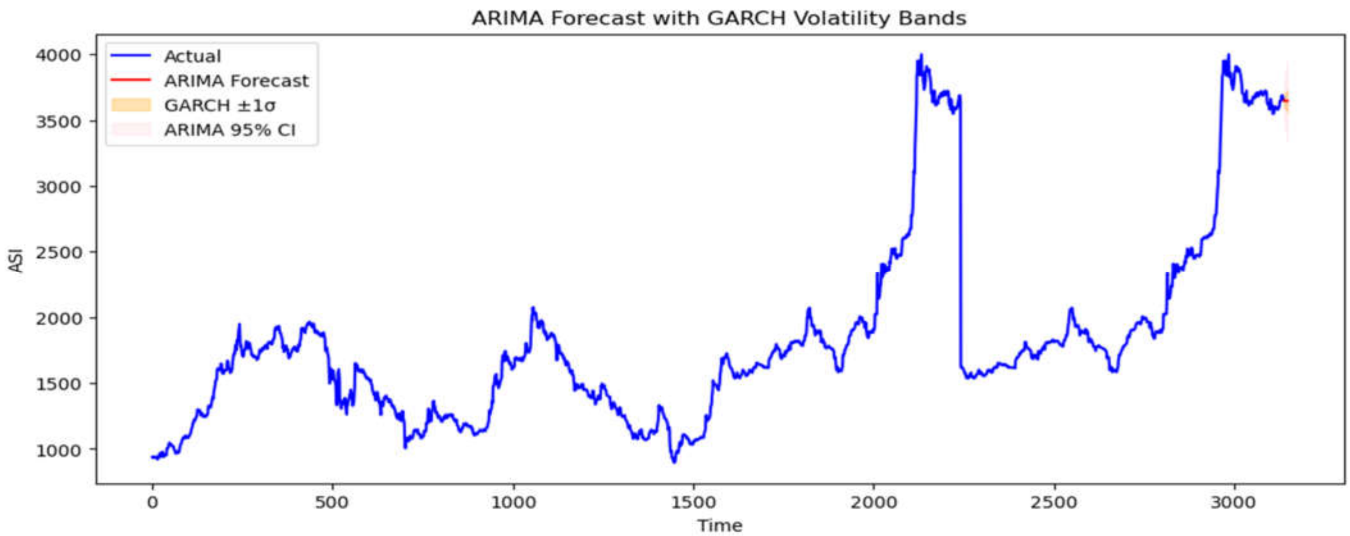

Figure 6: ARIMA forecast with GARCH (1,1)

Figure 6 and Table 3 present the combined ARIMA-GARCH forecasts for ASI, which include the mean forecast ($\hat{y}_{t+h}$), variance forecast (σ^2_{t+h}), and volatility forecast (σ_{t+h}). This approach provides both the expected ASI values and a measure of uncertainty around these predictions, accounting for the conditional variance dynamics captured by GARCH. So mathematically: $\hat{y}_{t+h} = E(y_{t+h} \mid \mathcal{F}_t)$. The variance forecast $\rightarrow \sigma^2_{t+h}$ and volatility forecast $\rightarrow \sigma_{t+h}$ and So: $\sigma^2_{t+h} = E(\epsilon^2_{t+h} \mid \mathcal{F}_t)$. The true ARIMA-GARCH forecast combines is given as : $y_{t+h} = \hat{y}_{t+h} \pm \sigma_{t+h}$. The results are shown in table 4 below

Table 4: Mean and conditional variance forecast

Period	Mean Forecast	Lower Bound	Upper Bound	Variance Forecast	Volatility (Std Dev)
3138	3648.5011	3611.6554	3685.3468	1357.6042	36.8457
3139	3647.6821	3603.4646	3691.8996	1955.1913	44.2175
3140	3647.1576	3596.6326	3697.6826	2552.7783	50.5250
3141	3646.8217	3590.6935	3702.9498	3150.3653	56.1281
3142	3646.6065	3585.3860	3707.8271	3747.9523	61.2205
3143	3646.4688	3580.5481	3712.3895	4345.5393	65.9207
3144	3646.3805	3576.0732	3716.6879	4943.1263	70.3074
3145	3646.3240	3571.8880	3720.7600	5540.7133	74.4360
3146	3646.2878	3567.9405	3724.6351	6138.3003	78.3473
3147	3646.2646	3564.1922	3728.3371	6735.8873	82.0725

Source: Authors computation (2026).

This methodology provides anticipated ASI figures along with a measure of the uncertainty surrounding these estimates, incorporating the dynamics of conditional variance highlighted by GARCH. According to Table 4, the average forecasts display a relatively stable trend, reflecting the model's assumption of a constant mean. Conversely, the forecasts for both variance and volatility increase throughout the forecast period, indicating heightened uncertainty regarding ASI values as the forecast horizon extends. The widening gap between the lower and upper limits illustrates the risks tied to future ASI variations, aligning with the significant persistence noted in the GARCH analysis. The mean forecast is relatively stable (~3646–3648), indicating a steady expected ASI level. The forecast interval (lower–upper bounds) widens over time, reflecting increasing uncertainty. The variance and volatility increase progressively, showing volatility clustering and persistence, consistent with the GARCH (1,1) results. This suggests that while the expected ASI remains stable, risk (uncertainty) grows over the forecast horizon.

Discussion of results

The application of the ARIMA(1,1,1) model to the first-differenced ASI data indicates that the current fluctuations in the index are affected by prior changes as well as earlier shocks. The AR coefficient implies that momentum from preceding periods significantly influences current ASI levels, while the MA coefficient captures the effects of past unexpected events. An analysis of the residuals, as illustrated in Figure 4, confirms that the ARIMA model successfully identifies the linear trends within the series, with residuals demonstrating randomness and lacking any recognizable pattern. The ARCH test outcomes presented in Table 3 further support this finding, showing no significant ARCH effects, thereby suggesting that the conditional variance of the residuals does not depend linearly on previous values. Nonetheless, the occurrence of volatility clustering in the ASI series necessitated the use of the GARCH (1,1) model to accurately capture the changing volatility over time.

The GARCH(1,1) model indicates that recent shocks substantially influence current volatility, evidenced by the elevated alpha parameter, while earlier volatility exhibits moderate persistence through the beta coefficient. The total alpha and beta equals one, indicating a behaviour akin to IGARCH, which suggests that shocks have enduring effects on future volatility. The long-term variance component points to a significant baseline level of volatility around which fluctuations take place, with the mean return being minimal and statistically insignificant. Figure 4 displays the conditional volatility obtained from the GARCH model, highlighting moments of increased risk. This visualization confirms the existence of volatility clustering, demonstrating that surges in ASI movements are succeeded by prolonged periods of heightened uncertainty, consistent with findings in the financial time series (Engle, 1982; Bollerslev, 1986, Deebom et al., 2023).

The joint ARIMA-GARCH predictions illustrated in Figure 5 and Table 4 outline not only the anticipated ASI figures but also the corresponding conditional volatility. The average forecast shows a consistent level throughout the assessed period, signifying the assumption of a constant mean in the model, while the variance and volatility projections increase steadily, indicating escalating uncertainty over time. This expansion of forecast ranges highlights that while the expected ASI trajectory appears stable, the risk surrounding future outcomes increases, stressing the importance of modelling conditional variance for forecasting financial metrics. Such trends of volatility clustering and enduring uncertainty align with previous research concerning stock market indices as well as financial data in emerging markets, which reveal that shocks often have lasting impacts and risk tends to build throughout forecast intervals. The findings affirm the necessity of utilizing ARIMA to capture the mean dynamics of the ASI and GARCH to portray the conditional volatility (Deebom. *et al,* 2023). The constancy of the mean forecast alongside the rising volatility accentuates the dual characteristics of the data series: although the anticipated index does not experience significant fluctuations, the surrounding uncertainty escalates, a crucial aspect for risk evaluation and planning in financial decision-making. This observation corroborates past empirical research highlighting the enduring nature of shocks in financial markets and the significance of incorporating heteroskedasticity to achieve more dependable forecasts.

5. Conclusion

The examination reveals that the ARIMA (1,1,1) model effectively captures the linear relationships present in ASI fluctuations, as evidenced by the random nature of the residuals. Although the ARIMA residuals do not display notable ARCH effects in isolation, the GARCH (1,1) model uncovers significant volatility clustering and persistence in shocks, suggesting IGARCH-like characteristics. This indicates that periods of heightened or decreased market volatility in the NGX are likely to persist, highlighting the necessity of factoring in conditional variance when making predictions. Projections exhibit a consistent anticipated ASI level, yet the ascending variance and broadening confidence intervals emphasize the escalating unpredictability and risks tied to future market scenarios. Overall, the integrated ARIMA-GARCH approach offers a robust framework for capturing both average behaviour and time-sensitive volatility in the ASI data, consistent with earlier research on emerging market indices where volatility persistence is a defining feature.

In terms of Implications: The outcomes of this study translate into several critical, practical applications for stakeholders in the Nigerian financial ecosystem: *(i)* For Investors and Portfolio Managers: Recognizing the IGARCH-like persistence means that a "shock" today (such as a sudden currency devaluation or oil price shift) will impact market volatility for an extended period. Practically, investors should avoid "buying the dip" during initial spikes in volatility until the GARCH model signals a decay in conditional variance. Furthermore, the broadening forecast intervals suggest that long-term static hedging is insufficient; managers should instead adopt dynamic hedging ratios that adjust as the ARIMA-GARCH confidence intervals expand. *(ii)* For Financial Analysts and Risk Officers: The findings provide a concrete basis for Value-at-Risk (VaR) recalibration. Analysts can utilize these predictions to set more realistic margin requirements and collateral haircuts. Because uncertainty escalates over time, analysts should use the model to perform "volatility stress tests," simulating how a persistent shock would impact the liquidity of specific equity portfolios over a 30-to-90-day horizon. *(iv)* For Policymakers and Regulators: The presence of volatility clustering justifies the implementation of macro-prudential "circuit breakers" on the NGX. Regulators can use the GARCH framework as an early-warning system; when conditional variance reaches a pre-defined threshold, proactive measures—such as temporary trading halts or stricter short-selling limits—can be enacted to prevent systemic contagion. *(v)* For Academic Advancement: This methodology underscores the importance of a dual-lens analysis (mean and variance). In emerging economies like Nigeria, where markets are often informationally inefficient, these findings suggest that future models must incorporate non-linear residuals. This opens the door for practical research into "Machine Learning-GARCH" hybrids that can better navigate the enduring impacts of fiscal and monetary shocks on equity prices.

The following outlines the strategic recommendations for the Nigerian Exchange (NGX) based on evidence of volatility clustering and market inefficiency, organized by stakeholder groups:

i. For investors and portfolio managers, the primary objective should be a transition toward volatility-aware systems. This begins with implementing hybrid ARIMA-GARCH frameworks, which move beyond basic moving averages by using ARIMA to capture linear mean returns while employing GARCH (1,1) models to forecast a "volatility envelope." In practice, NGX fund managers can use these insights to automate risk management; for instance, when the GARCH component signals rising conditional variance, stop-loss thresholds should be tightened to protect capital. Furthermore, rather than relying on static annual data, professionals should utilize 180-day rolling windows for a "walk-forward" optimization approach. This allows models to capture recent structural shifts in the Nigerian macro-environment while discarding outdated shocks. Finally, risk thresholds should be refined by calibrating Dynamic Value-at-Risk (VaR) through Filtered Historical Simulation, ensuring that capital requirements expand realistically during periods of market turbulence.

ii. Market regulators, specifically the Nigerian Securities and Exchange Commission and the Nigerian Exchange Group, must adopt more proactive oversight mechanisms to stabilize the exchange. A critical step is the deployment of volatility-based monitoring via an automated "Persistence Shock" dashboard. By tracking conditional variance, regulators can trigger algorithmic circuit breakers or mandatory "cooling-off" periods whenever volatility exceeds a 3-standard-deviation threshold, effectively preventing panic-induced feedback loops. To push the NGX closer to Weak-Form Efficiency, regulators should also prioritize information symmetry by enforcing stricter T+1 disclosure mandates for material news. Minimizing the time lag between corporate events and public reporting reduces the window in which historical price patterns can be exploited for abnormal gains, thereby fostering a more transparent trading environment.

iii. Academic researchers and data scientists are encouraged to evolve existing modelling techniques to account for the NGX's unique complexities. Future studies should expand into the ARIMAX-GARCH domain by treating the market as an open system, integrating exogenous variables (X) such as Brent Crude prices and NGN/USD exchange rates to understand how global energy and currency shocks spill over into local equities. Beyond traditional econometrics, there is a significant opportunity to develop deep learning hybrids, such as CNN-LSTM-GARCH architectures. By utilizing Long Short-Term Memory (LSTM) networks to capture long-range dependencies and non-linear residuals, researchers can "clean" market signals of noise. This integration of machine learning with traditional GARCH outputs offers the most promising path toward improving the accuracy of multi-day price point forecasting in a volatile market

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Appendix

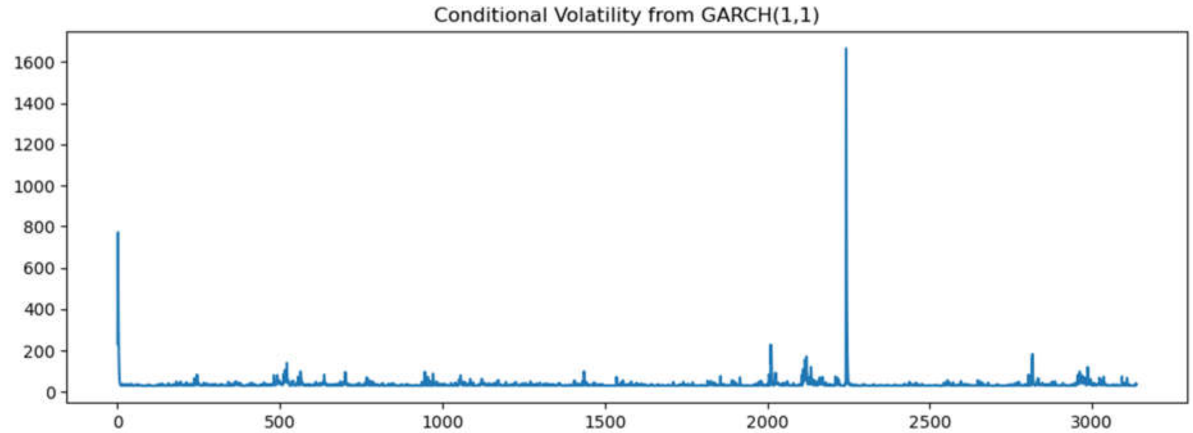
ARIMA Model Summary:

```
                     SARIMAX Results
==============================================================================
===
Dep. Variable:                  ASI   No. Observations:               3138
Model:               ARIMA(1, 1, 1)   Log Likelihood            -16192.264
Date:              Sat, 21 Mar 2026   AIC                        32390.528
Time:                      14:44:01   BIC                        32408.681
Sample:                           0   HQIC                       32397.043
                             - 3138
Covariance Type:                opg
==============================================================================
===
            coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1       0.6404      0.099      6.456      0.000       0.446       0.835
ma.L1      -0.5731      0.102     -5.619      0.000      -0.773      -0.373
sigma2   1782.1796      1.612   1105.243      0.000    1779.019    1785.340
```

```
==========================================================================================
Ljung-Box (L1) (Q):                   0.21   Jarque-Bera (JB):         369191499.04
Prob(Q):                              0.64   Prob(JB):                         0.00
Heteroskedasticity (H):              11.66   Skew:                           -34.85
Prob(H) (two-sided):                  0.00   Kurtosis:                      1682.19
==========================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

ARCH Test on ARIMA Residuals: F-statistic=0.0470, p-value=1.0000
Iteration:      5,   Func. Count:     28,   Neg. LLF: 17725.18984283944
Iteration:     10,   Func. Count:     55,   Neg. LLF: 16055.143131451141
Iteration:     15,   Func. Count:     80,   Neg. LLF: 16017.100225565684
Iteration:     20,   Func. Count:    105,   Neg. LLF: 16008.164055627007
Optimization terminated successfully    (Exit mode 0)
            Current function value: 16008.16405510159
            Iterations: 21
            Function evaluations: 110
            Gradient evaluations: 21

GARCH(1,1) Summary:
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -16008.2
Distribution:                  Normal   AIC:                           32024.3
Method:            Maximum Likelihood   BIC:                           32048.5
                                        No. Observations:                 3138
Date:                Sat, Mar 21 2026   Df Residuals:                     3137
Time:                        14:44:01   Df Model:                            1
                               Mean Model
============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
----------------------------------------------------------------------------
mu            10.1647      6.407      1.586      0.113 [ -2.393, 22.722]
                              Volatility Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
omega        597.5870    302.674      1.974  4.834e-02 [  4.357,1.191e+03]
alpha[1]       0.6714      0.276      2.430  1.510e-02   [  0.130,  1.213]
beta[1]        0.3286  5.390e-02      6.096  1.086e-09   [  0.223,  0.434]
```

==

Covariance estimator: robust



GARCH Forecast for next 10 periods:

	h.01	h.02	h.03	h.04	h.05 \
3137	1357.604244	1955.19125	2552.778256	3150.365261	3747.952267

	h.06	h.07	h.08	h.09	h.10
3137	4345.539273	4943.126278	5540.713284	6138.30029	6735.887296